TITLE: BUSINESS BULLSEYE

Student/Class Goal
Students often have to make financial decisions based on predictions of future outcomes.

Outcome (*lesson objective*)
Given either the factored or expanded form of trinomials, students will set up the other form with 100% accuracy. Students will solve, graph, and interpret quadratic functions in the context of basic profit modeling.

Time Frame

2 hours

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 5-6

Number Sense	**Benchmarks**	**Geometry & Measurement**	**Benchmarks**	**Processes**	**Benchmarks**
Words to numbers connection	5.1, 6.1	Geometric figures		Word problems	5.25, 6.26
Calculation		Coordinate system	5.7. 6.7	Problem solving strategies	5.26, 6.27
Order of operations		Perimeter/area/volume formulas	5.8	Solutions analysis	5.27, 6.28
Compare/order numbers		Graphing two-dimensional figures		Calculator	5.28, 6.29
Estimation		Measurement relationships		Math terminology/symbols	5.29, 6.30
Exponents/radical expressions		Pythagorean theorem		Logical progression	5.30, 6.31
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	**5.31, 6.32**
Patterns/sequences		Measurement conversions		Mathematical material	5.32, 6.33
Equations/expressions	5.16, 6.16	Rounding		Logical terms	
Linear/nonlinear representations	**5.17, 6.17**	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing	5.18, 6.18	Data interpretation		Real-life applications	5.35, 6.36
Linear equations		Data displays construction		Independence/range/fluency	5.36, 6.37
Quadratic equations	**5.18, 6.20**	Central tendency			
		Probabilities			
		Contextual probability			

Materials
Algebra Tile Task Sheet
Yarns to Yearn For Task Handout
Mammalian Threads Task Handout
Farmer's Market Task Handout
Teacher Answer Sheet
Vocabulary Sheet

Learner Prior Knowledge

Students should be able to manipulate single-variable **algebraic equations** and two-variable **algebraic expressions**.

Instructional Activities

Step 1: *Review*. Draw a rectangle on the board with the dimensions 5 inches x 6 inches. Ask students what the area is. Now, draw another rectangle with the dimensions (3 inches + 2 inches) x (1 inch + 5 inches). Ask students how they might approach this without adding the 3 and the 2 or the 1 and the 5. If there are no correct answers, proceed by sectioning off the shorter side into 3 and 2, and the longer side into 1 and 5. Then construct the four smaller rectangles inside the larger one (3 x 1), (3 x 5), (2 x 1), (2 x 5). Calculate each of these four areas and show that the sum equals the same as the original. In Step 2, we will extend this process to the abstract.

Teacher note for Steps 2–4. If it appears that these problems are too simple for the level your students are at, make them more challenging by giving them the final areas and having them find the original factors. For example, in Step 2, you would give them 1"x^2" tile, 4 "x" tiles, and 3 "1" tiles. They would then have to arrange these into one large rectangle to discover the (x + 3) and the (x + 1).

Step 2: (I do) *Teacher models the solution process.* Write the following expression on the board: (x + 3)(x + 1) Pass out an algebra tile set to each student and explain what each tile represents. Show how to construct two adjacent sides of a rectangle, with length (x + 3) and width (x + 1). Immediately divide it into the four smaller rectangles, as you did in Step 1, trying your best to keep the x roughly the same length on each side. Show how you can calculate the area of each of the four smaller rectangles by the basic rectangle formula (length x width). Now multiply this same product out using the **FOIL method** (First – Outer – Inner – Last).

Step 3: (we do) *Teacher and students collaboratively work through the problem.* Write the following expression on the board: (x + y + 1)(x + 2). Comment to yourself that the FOIL method doesn't seem to work because it doesn't account for the y. Ask students how they could use the algebra tiles to find the product. Prompt them as necessary, so that they construct a rectangle and label one side x + y + 1, and the adjacent side x + 2. Then, divide the first side into x, y, and 1, and the second side into x and 2. Again, try to keep the x's roughly the same length so that the x^2 turns out to look like a square.

Step 4: (you do) *Students independently work through the problem.* Ask students to expand the following product: (2x+2)(y+1). They may break the 2x into x + x, or keep it together; if two students approach it differently, ask both to share their method with the whole group.

Step 5: Pass the Algebra Tile Task sheet out to the students. Work through the first two problems together with students (so that they understand how the negative tiles work and how to factor) and then allow them to work through the rest of the problems in pairs.

Step 6: *Introduce alternate representations*. Ask students how they could solve $x^2 + 2x + 2 = 0$ using their algebra tiles (note: they won't be able to factor the expression on the left-hand side of the equation). Explain how the advantage of the algebra tiles is that they show the connection between geometry and algebra. The disadvantage is that the tiles are very limited in the range of problems they can be used for solving (not to mention the fact that the students will not be allowed to use them in their standardized exam!) This is where algebra techniques come in handy. Solve this problem with the students by completing the square, by quadratic formula, and by graphing it (if necessary, refer to the *Teacher Answer* sheet for these three processes). If this seems to be a review for students, continue on to Step 7. If students struggle with any of these three processes, use explicit instruction (I do, we do, you do) with a few simple examples until they are comfortable moving on.

Step 7: Introduce the business context. Begin by defining the **cost function** as C(x), where x is the amount of items you are purchasing. Give the example of ordering books online for $20 each plus a set $5 handling fee. In this case, C(x) = 20x + 5. Point out the difference between the two parts of the equation; the first part increases with every extra book you buy, whereas the 5 is not dependent on the x. Next, introduce the **revenue function** as R(x), where x is still the amount of items you purchase. Tell students that revenue functions generally have a y-intercept of 0. In other words, although the general quadratic form is $f(x) = ax^2 + bx + c$, this c is usually 0 in a revenue function. Point out that this is logical, because if you do not purchase any items (x=0), you should not expect to make any revenue from selling items. Finally, introduce the **profit function** as P(x) = R(x) – C(x). In other words, your profit is the amount of money you receive minus your total costs. When your costs equal the revenues, there is no profit (if R(x) = C(x), P(x) = 0). This is known as an equilibrium point because the costs and revenues are balanced. The other important point is the point of maximum profit. Give students an example by drawing a parabola opening down with profit on the y-axis and amount of items purchased on the x-axis. Point out that the vertex of the parabola is the place of maximal profit.

Step 8: (I do) *Teacher models the solution process through the Yarns to Yearn For Task.* You will want a calculator for Step 8; in the following two steps, a calculator is not necessary, but can be used to check the answer. The revenue, cost, and profit functions are labeled with capital letters to help students remember what they represent. However, if these get too confusing for students, you can always change R(x), C(x), and P(x) to y1, y2, and y3, respectively. When you use the talk aloud process to solve this problem, make sure you talk through the setup of the cost equation and how you know that the 45 gets attached to an x (because it is dependent on number of items), but the 200 does not. When you find the break-even points, it will probably be easier to use the quadratic formula than to complete the square.

Step 9: (we do) *Teacher and students collaboratively work through the Mammalian Threads Task*. The first four questions are quite similar to the procedure in Step 8, so call on student input as much as possible before taking initiative yourself. Question #5 will require a bit more thought, but it should follow easily if you can graph it on the same graph as #4. In addition to answering the question, you may want to look at how the two profit functions compare for various amounts of sweaters purchased.

Step 10: (you do) *Students independently work through the Farmer's Market Task*. Depending on your class dynamics, either partner students together or have them work individually. Before you pass out the task, explain that you want the students to tackle this problem as independently as possible. After passing out the handouts, walk around the room silently monitoring the students' progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the first two tasks. Question #5 is very similar to #5 in Step 9, but students should be able to explain why, in this case, the optimal number of kilograms didn't change. If they do not make this observation on their own, ask them during the discussion in Step 11.

Step 11: Have each student (or pair) share both the process they used and their final comparisons. When students disagree, do not immediately provide the correct answer; allow each student or pair to try to convince the other first.

Assessment/Evidence (*based on outcome*)
Steps 10 and 11 will serve as evidence of student mastery. During Step 10, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students speak out loud as they solve the problem. During Step 11, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions.

Exit Slip: 1. Expand (2x+1)(y-2) using algebra tiles.
2. What is the maximum profit that can be made if $R(x) = x^2 - 6x + 4$ and $C(x) = 2x + 2$? (Answer: -$4, or a loss of $4; this would not be a wise business venture!)

Teacher Reflection/Lesson Evaluation
Not yet completed.

Next Steps
Have students choose one item that they try to save money on by buying in bulk. Ask them to think about how this profit model is similar or different to the decision they make of *how much* bulk to buy in. (In other words, if they buy canned soup in bulk when it is on sale, how do they decide how many cans to buy, and how is this similar to the Business Bullseye lesson?)

Technology Integration

The National Library of Virtual Manipulatives site is filled with tested activities and explorations for understanding mathematics. The link below will take you through several exercises in using Algebra Tiles, which you can then try with your students using the classroom sets (of real tiles).
http://nlvm.usu.edu/en/nav/frames_asid_189_g_4_t_2.html?open=activities&from=category_g_4_t_2.html

This pdf gives a more detailed explanation of the business concepts and mathematical applications from this lesson.
http://math.boisestate.edu/mlc/ACT/ElemEd10.pdf

Purposeful/Transparent
Quadratic functions underlie many processes in our everyday lives ranging from physics (trajectory of objects) to business (cost-revenue function). Key to the study of quadratics is optimization – finding a maximum or minimum point. Since students would want to maximize their profit in financial decisions, this lesson focuses on interpreting cost, revenue, and profit functions.

Contextual
Although this lesson uses a simplified version of the profit modeling conducted by actual businesses, the basic principles are the same. Determining the optimal purchase amounts is one of the key points of success to any business, and can be applied to family business management in areas like grocery shopping or party planning.

Building Expertise
By using algebra tiles, students extend their knowledge of algebraic representations to geometric representations of quadratic functions. They then apply this understanding to quadratic profit equations, and interpret the key aspects of quadratics in common language.

Algebra Tiles Task Sheet

Using the algebra tiles, complete the following:

1. Expand. $(x-1)(y+3)$
2. Factor. $x^2 + 3x + 2$
3. Expand. $(2x-1)(y+5)$
4. Factor. $x^2 + 7x + 10$
5. Factor. $xy - y + 3x - 3$
6. Factor. $2x^2 + 5x + 2$

Business Bullseye: Vocabulary Sheet

Algebraic equation – an equation that includes at least one unknown variable.

Algebraic expression – an expression that includes at least one unknown variable. Note that an algebraic equation is two algebraic expressions set equal to each other. Thus, $x + 3 = 7$ would be an algebraic equation, but $x + 3$ would be an algebraic expression. .

Cost function – a function of x, where x is the amount of items being purchased. $C(x)$ is used to represent the total item cost.

FOIL method – a process for expanding a product of binomials. The acronym stands for Firsts-Outers-Inners-Lasts. If the product is in the form $(a + b)(c + d)$, the "Firsts" are a and c; the "Outers" are a and d; the "Inners" are b and c; and the "Lasts" are b and d.

Profit function – a function of x, where x is the amount of items being purchased. $P(x)$ is used to represent the total expected profit if x items are purchased. $P(x)$ can be obtained by subtracting $C(x)$ from $R(x)$.

Revenue function – a function of x, where x is the amount of items being purchased. $R(x)$ is used to represent the total expected revenue if x items are purchased.